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Alpha-decay without tunnelling : Alpha-decay radius of even-even alpha-emitters

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Following the *α-decay without tunnelling* approach, α decay radius R, daughter radius R_d and daughter radius-constant r_0 of trans-lead even-even α-emitters have been extensively computed. The values of R and r_0 lie in the ranges of 9·4–11·2 fm and 1·23–1·46 fm., respectively. Present expression for the decay-radius demonstrates in explicit terms that fluctuations of decay-radii, also observed in the Gamow picture, are due to the finite α-pre-formation probability. Radii of heavy nuclides extracted from scattering and fission-spallation reaction cross-sections find consistent places in the present scheme. The present study lastly shows that α-decay in a deformed nuclide takes place preferentially along the axis of symmetry of the nuclide in order to encounter the shortest coulomb-barrier height

1 INTRODUCTION

Since the very birth of Gamow theory of α-decay, α-preformation probability and α-decay radius of the decaying nucleus have been engaging the attention of workers in this field. These two nuclear parameters are of crucial importance to the understanding of the α-decay process. α-preformation is directly related to the structure of the decaying nucleus; α-decay radius is, on the other hand, a measure of its size as defined by the competition between the attractive nuclear force and repulsive coulomb force experienced by the α-particle in the process of its formation and emission. While a clear picture of α-pre-formation probability is gradually emerging (Basu 1976, 1977), present knowledge of α-decay radii is, in spite of volumes of works on these, still in an unsatisfactory state (Perlman & Rasmussen 1957).

The quantum-mechanical explanation of the α-decay process by Gamow is represented by the following relation.

$$\lambda = f.P \qquad \text{.. (1)}$$

where λ = total α-decay rate of the α-emitter

f = frequency factor,

and P = penetrability.

As is well-known, f and P contain in them the α-preformation probability implicitly and α-decay radius explicitly, respectively, of each α-emitter. Since the birth of Gamow theory, the primary concern of all workers of this field has been to extract α-preformation probability and α-decay radius of different α-emitters from the exploitation of relation (1). Relation (1) is again a two-parameter expression and hence it is an impossible task to try to know these two unknown parameters, α-pre-formation probability and α-decay radius, from this relation with the help of the knowledge of experimental decay-rates only. It is exactly this impossible task that workers in this field have all along been trying to achieve. The usual practice is to fix f at some ad-hoc value for all the even-even α-emitters and then to compute α-decay radii of different α-emitters from the experimental values of λ with the help of relation (1). Obviously fixation of f at an ad-hoc single value not only implies a single α-preformation probability for all the even-even α-emitters contrary to the structure concept of a nucleus, but also means complete loss of information about α-preformation probability of different nuclei (Basu 1976, 1977). In this approach, f has been assigned by different workers different values in the wide range of ($\sim 10^{15}$sec^{-1} - 10^{21}sec^{-1}). Majority of the workers (Biswas & Patro 1948, Kaplan 1951, Perlman & Ypsilantis 1950, Asaro 1953, Perlman *et al.* 1950) assumed values of f in the range $\sim 10^{20}$–10^{21} sec^{-1} and calculated the decay radius of different nuclides by treating it as a freely adjustable parameter in the one-body Gamow relation (1) or in different versions of it. Values obtained by them for different even-even α-emitters are between 8 and 9·6 fm. Bethe (1937), on the other hand, chose a drastically low value of the frequency factor at $\sim 10^{15}$ sec^{-1}. It is not surprising that his many-body values of decay radii are between 11·3 and 13·2 fm., about 40% higher than the values obtained by the workers already referred to and hence treated with reservations. This wide discrepancy in the computed value of the decay radius for a single nuclide as well as the discrepancy in the ranges of values for different nuclides is evidently a consequence of the differing choices of the value of the frequency factor and has resulted in a confusing state of affairs in the understanding of the significance of the term, α-decay radius. Besides this, the decay radius evaluated in this approach is invariably identified by all workers with the radius of the daughter or, at times, with the daughter nucleus radius plus a reduced α-particle radius, quite contrary to the true import of the term 'α-decay radius', which has been significantly defined by Blatt & Weisskopf (1952) and Hanna (1959) as the sum of the radii of the daughter nucleus and the free alpha-particle. In short, discrepancy in the one-body decay radius values, their non-conformity to the proper definition and the complete loss of information about α-preformation probability have resulted in a really confusing situation and hence call for serious reservations in dealing with these radius values. These values are again all the more unreliable in view of the fact that the different chosen values of f are uncertain due to lack of knowledge of internal details.

Alpha-decay is a unique nuclear phenomenon which probably makes α-decay radius also unique in character in the sense that the qualitative and quantitative significance of the decay-radius cannot be rightly assessed by comparing it with other independent radius-determinations. This is because other independent determinations explore aspects of the nuclear size quite different from that explored by the α-decay process. Only elastic and inelastic scattering of α-particles by and α-fission-spallation reactions in heavy nuclides (Rasmussen 1958) can simulate the α-decay process and as such, radii evaluated from these cross-sections should be comparable to the α-decay radii of these nuclides. Unfortunately, one-body decay-radii, except the Bethe ones are much shorter than these radius determinations. Bethe values are again too large to be taken seriously. One-body model dependent α-decay radii are, in short, unreliable and present knowledge about these is consequently in a very unsatisfactory state. As pointed out by Perlman & Rasmussen (1957), one can hope of obtaining reliable α-decay radii only when fundamental intranuclear mechanism of α-formation is well understood.

Recently Basu (1974) has made phenomenologically an *ab initio* investigation of the α-decay problem. With no assumptions whatsoever, he has developed a model of α-decay which describes in clear terms how the last $2n$ $2p$ system inside the alpha-emitter is formed into an alpha-particle and how, in the process of α-formation, the coulomb-barrier is crossed over and the nascent α-particle finds itself at the top of the barrier wherefrom it can easily escape without tunnelling. As a happy consequence of this model follow a simple but accurate expression of absolute α-preformation probability for an even-even α-emitter and a formula for the α-decay radius quite in conformity with the proper definition of this term. All these findings and their consequences have been detailed in subsequent papers (Basu & Sen 1975, Basu 1976).

The purpose of this article is to present an extensive tabulation of α-decay radii of trans-lead even-even α-emitters calculated in the present approach, and to discuss their conformity to the proper definition and their reliability. Some features in the decay radii previously observed but unexplained by the one-body model will also form a subject of close investigation. Some comments on α-decay in deformed nuclides will also be made.

2 Theory

One comes across the following expressions for α-decay radius R and α-performation probability P_α in the model of 'α-decay without tunnelling' (Basu 1974)

$$R \quad R_0(1+P_\alpha) \tag{2}$$

and

$$P_\alpha \quad {E_{C_1}}^\alpha/B(\alpha)$$

where

$$R_0 = 2(Z-2)e^2/(\quad)B(\alpha)$$

$B(\alpha)$ = Binding energy of a free α-particle, and

${E_{C_1}}^\alpha$ = Effect. α-clustering energy of the last $2n$-$2p$ syst   de the parent α-emitter.

One can compute simultaneously the α-decay radius and the α-pre-formation probability of the even-even α-emitters from relations (2) and (3) respectively, evidently with the greatest accuracy unlike in the one-body approach in which all information about α-preformation is lost and the α-decay radius is calculated by treating it as an adjustable parameter. $E_{C_1}{}^{\alpha}$ can be evaluated accurately from the mass-relation due to Basu (1972) with the help of the mass-table. It should be noted here that relations (2) and (3) are obtained from considerations of fundamental intra-nuclear mechanism of α-formation and α-emission According to the prescription of Perlman & Rasmussen (1957), relation (2) would yield reliable α-decay radii

As has been clearly shown in previous work (Basu & Sen 1975), R in relation (2) conforms well to the proper definition of the α-decay radius so that one can split R in the following way

$$R = R_d + r_\alpha \qquad \ldots \quad (4)$$

where

R_d = Radius of the daughter nucleus, and

r_α = Radius of a free α-particle

= 2·08 fm. (Hofstadter 1956)

3. Results and Discussion

Values of α-decay radii evaluated from relation (2) for all the even-even α-emitters under consideration are given in column (2) of table 1. The values lie in the range of 9·4–11·2 fm. which is in between the Bethe values and the values due to Devaney (1953) and others. This suggests that correct decay-radii in conformity with the proper definition of the term may be obtained from the one-body model if one choses the value of the frequency factor in between $\sim 10^{15}$ sec^{-1} and $\sim 10^{21}$ sec^{-1} But in view of the close connection between the decay-radius and α-preformation probability as expressed by eq. (2), the validity of one-body approach to decay-radius becomes all the more open to question. It is again for the first time in α-decay studies that such an inter-connection between R and P_α is revealed. Needless to mention here that this revelation will greatly facilitate different model-dependent investigations in which the value of the α-decay radius, characteristic of the α-emitter in question, invariably plays a critical role.

Column 3 of table 1 gives values of radii of the doughter nuclei obtained from eq (4). The values are between 7·30 and 9·15 fm and the upper part of this range of values accommodates the α-decay radius (identified as the radius of the daughter nucleus) values obtained by Devaney, and others from the one-body model. This emphasizes, on the one-hand, that the present approach is quite consistent and points out, on the other hand, that the identification of the α-decay radius with the residual nucleus radius is not justified in the one-body model.

Table 1. Values of α-decay-radius R, daughter-radius R_d and daughter radius-constant r_0 for all even-even α-emitters $r_\alpha = 2{\cdot}08$ fm. (Hofstadter)

Parent nucleus	α-decay radius R fm	Daughter Nucleus R_d fm	r_0 fm
$^{216}Po_{84}$	9·47	7·39	1·24
^{214}Po	9·74	7·66	1·29
^{212}Po	9·67	7·59	1·28
$^{210}Em_{86}$	9·94	7·86	1·31
^{218}Em	10·23	8·15	1·36
^{222}Em	9·41	7·33	1·24
^{220}Em	9·75	7·67	1·30
^{208}Em	9·90	7·82	1·33
$^{224}Ra_{88}$	10·25	8·17	1·35
^{222}Ra	10·61	8·53	1·42
$^{232}Th_{90}$	10·64	8·56	1·40
^{230}Th	10·59	8·51	1·39
^{228}Th	10·31	8·23	1·35
^{226}Th	10·43	8·35	1·38
^{224}Th	10·5[illegible]	8·47	1·40
$^{236}U_{92}$	10·25	8·34	1·33
^{234}U	10·63	8·55	1·39
^{232}U	10·54	8·46	1·39
^{230}U	10·72	8·64	1·42
^{228}U	10·77	8·69	1·43
$^{242}Pu_{94}$	10·56	8·48	1·37
^{240}Pu	10·23	8·15	1·32
^{238}Pu	10·68	8·60	1·39
^{236}Pu	10·34	8·26	1·34
^{234}Pu	10·80	8·72	1·42
$^{244}Cm_{96}$	10·41	8·33	1·34
^{242}Cm	10·64	8·56	1·38
^{240}Cm	10·45	8·37	1·35
$^{252}Cf_{98}$	10·95	8·87	1·41
^{250}Cf	10·88	8·80	1·40
^{248}Cf	10·69	8·61	1·37
^{246}Cf	11·08	9·00	1·44
$^{254}Fm_{100}$	10·85	8·77	1·39
^{252}Fm	10·95	8·87	1·41
^{250}Fm	11·23	9·15	1·46

Present values of daughter-radius constant r_0 are shown in the last column of table 1. r_0 was calculated, following other workers, from the relation $R_d = r_0 A^{1/3}$ where A = mass-number of the residual nucleus. r_0 in the present approach lies in the range of 1·23–1·46 fm. while one-body model-dependent r_0's due to Devaney and others are between 1·25 and 1·58 fm. Evidently these two sets of values are more or less in good agreement and conform to the concept of incompressibility of nuclear matter. r_0 values due to Asaro (1953) show a decreasing trend with increasing Z quite contrary to the trend of present average

r_0 values which exhibit a slight increasing tendency with increasing Z. r_0 in the present approach does not show a constancy but exhibits, rather, fluctuations from one nuclide to another in each isotopic series. These fluctuations have their origin in the finite α-pre-formation probability as will be explained presently.

It is almost always the usual practice with various workers in this field to fit the computed decay-radii with the relation $R = r_0 A^{1/3}$. But this practice is probably not legitimate (Hanna 1959) as it ignores shell-effects or other structure-effects that may naturally be present in the decay-radius. Present expression-(2) for α decay radius clearly shows that the decay-radius explicitly depends on the α pre-formation probability P_α. P_α has, in fact, been found by Basu (1976) and Colli-Milazzo *et al* (1975) to exhibit distinct shell and sub-shell closure effects, and deformation effects in open-shell nuclei. In view of this dependence on α-preformation probability and deformation effects, no attempt has been made in the present approach to fit the computed decay-radii with the $A^{1/3}$-law. Such an attempt would, in fact, be incompatible with the fact that most α-emitters are deformed in shape. Obviously enough, fluctuations in the present r_0's, as pointed out earlier, are a simple case of inheritance of the fluctuations of α-decay radius due to finite α-preformation probability.

α-Decay radii in the Gamow picture, when fitted with the $A^{1/3}$-relation, exhibit clear fluctuations around the $A^{1/3}$-law (Hanna 1959). This is also the case with the present decay-radii. These fluctuations were supposed, in the one-body model, to be due to the finite α-pre-formation probability and non-spherical shape of the nuclide. It is really gratifying to note that it is only in the present approach that one finds that the α-decay radius is an explicit function of the α-pre-formation probability P_α which, in even-even isotopic series, is fluctuating in nature. It is now easy to appreciate that observed fluctuations of R are due to finite P_α and deformation effects present in P_α.

As has already been pointed out in the introduction one-body model-dependent decay-radii fall far short of radius values of heavy nuclides extracted from elastic and inelastic α-scattering cross-sections and fission-spallation reaction cross-sections in these nuclei (Rasmussen 1958). But in view of the near similarity of physical situations that come into play in these processes, the former radius values are expected to agree with the latter ones. This discrepancy still remains unexplained by the one-body model. It is, however, satisfying to note that radii of heavy nuclides obtained from above-mentioned cross-section measurements find, at long last, a consistent place in the present set of α-decay radii and there exists no discrepany among the radius values of these two different approaches.

It should be again pointed out here that present values of R_d (and hence R and r_0) were obtained from considerations of fundamental process of α-formation and α-emission, and not by treating any of these as an adjustable parameter unlike in the one-body model. It is, therefore, natural that these values should prove very consistent and reliable as has been observed in the foregoing discussions.

Most of the α-active nuclei have stable spheroidal deformation which is definitely reflected in the mechanism of α-formation and α-emission and consequently in the α-decay radius. The expression-(2) for the α-decay radius is similar to the expression for the semi-major axis of an ellipse with an eccentricity proportional to P_γ. This similarity clearly points to an important aspect of the α-decay phenomenon. If the parent nucleus is permanently deformed, then formation and emission of an α-particle take place preferentially along the axis of symmetry of the deformed nuclide in order to experience the shortest height of the coulomb-barrier. But if the α-emitter is not permanently deformed, then the $2n$–$2p$ cluster, in the process of its transition into an α-particle, so deforms the nucleus as to be able to decay preferentially along the axis of symmetry in order to encounter the least coulomb-barrier height. The present expression for the alpha-decay radius of even even nuclei for the case of ground-state transition substantiates the suggestion by Hill & Wheeler (1953) that, in the Gamow picture, α-particle tunnelling from a deformed nuclide in the ground-to-ground state transition will be preferentially in a direction where the potential barrier is thinnest i.e. in the case of a prolate nucleus in the direction of the nuclear axis. In short, the effect of increasing prolate deformation would be to increase the barrier-penetrability very markedly, or equivalently decrease the barrier-height appreciably.

4 Conclusion

Highly idealized nature of the assumed potential, the assumption of a sharp cut-off radius and its wrong identification with the residual nucleus radius and, last but not the least, complete ignorance of fundamental mechanism of α-formation deprive the one-body model-dependent decay-radii of their proper quantitative and qualitative significance. Another major drawback in the one-body approach to α-decay radius investigation is the lack of appreciation of the very intimate connection between the decay-radius and α-preformation probability as revealed by the present study. The model of *α-decay without tunnelling* is, on the other hand, based only on considerations of intra-nuclear mechanism of α-formation and α-emission. It is, therefore, not surprising to note that α-decay radii in this *α-decay without tunnelling* approach prove, as has been amply demonstrated in the foregoing discussions of even-even nuclei, very consistent, very reliable and strictly conforming to the proper definition of the term, 'α-decay radius'. Observed fluctuations of decay-radii were speculated in the one-body model to be due to finite α-preformation probability and deformation effect; but it is only in the present approach that these observatons can be explained in clear and explicit terms as due to the aforesaid causes. Again large radii of heavy nuclides extracted from α-elastic and α-inelastic scattering cross-sections and fission-spallation reaction cross-sections find consistent places in the present scheme but appear incongruous in the one-body model. Present investigation clearly brings out the fact that α-decay in deformed nuclides takes place preferentially along the axis of symmetry of the nucleus in order to encounter a least coulomb-barrier height. In short, the present study marks out directly the present approach to the investigation of α-decay radii as a very powerful and successful one, and points indirectly to the firm foundation of the model of *α-decay without tunnelling*.

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